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Dynamic response of an Ising system to a pulsed field

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The dynamical response to a pulsed magnetic field has been studied here both using Monte Carlo simulation and by solving numerically the mean-field dynamical equation of motion for the Ising model. The ratio R_p of the response magnetization half-width to the width of the external field pulse has been observed to diverge and pulse susceptibility χ_p (ratio of the response magnetization peak height and the pulse height) gives a peak near the order-disorder transition temperature T_c (for the unperturbed system). The Monte Carlo results for the Ising system on a square lattice show that R_p diverges at T_c, with the exponent $\nu z \cong 2.0$, while χ_p shows a peak at T_c^e, which is a function of the field pulse width δt. A finite-size (in time) scaling analysis shows that $T_c^e = T_c + C(\delta t)^{-1/x}$, with $x = \nu z \cong 2.0$. The mean-field results show that both the divergence of R and the peak in χ_p occur at the mean-field transition temperature, while the peak height in $\chi_p \sim (\delta t)^y$, $y \cong 1$ for small values of δt. These results also compare well with an approximate analytical solution of the mean-field equation of motion. [S1063-651X(97)02903-6]

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I. INTRODUCTION

The dynamic response of the Ising systems has recently been studied extensively by employing computer simulations [1]. In particular, the study of dynamical response of Ising systems to an oscillating magnetic field [2,3] has led to many intriguing dynamic phenomena, such as dynamic hysteresis and the fluctuation-induced dynamic symmetry-breaking transitions in (low, e.g., one-, two-, or three-dimensional) the Ising system in the presence of an oscillating field. Acharyya and Chakrabarti also noted [3,4] some anomalous behavior in the growth of pulse susceptibility, in Ising systems under pulsed magnetic fields of finite durations.

Usually, when a cooperatively interacting thermodynamic system in equilibrium is perturbed (with the perturbation having a step-function-like variation with time), the relaxation of the system (to the equilibrium appropiate to the perturbed state) is observed to follow the common Debye-type form with a single relaxation time. The standard (Debye) form for any response function (say magnetization of the Ising system) $m(t)$ is

$$m(t) \sim m(\infty) + A\exp(-t/\tau), \tag{1.1}$$

where τ is the relaxation time, $m(\infty)$ denotes the new equilibrium value, and A is a constant. As the critical temperature is approached τ shows a critical slowing down; τ diverges at the critical temperature T_c:

$$\tau \sim \xi^z \sim (T - T_c)^{-\nu z}, \tag{1.2}$$

where ξ is the correlation length, z is the dynamic exponent, and ν is the correlation length exponent [1].

Here we have investigated in detail the response of pure Ising systems to pulsed magnetic fields of finite duration, using Monte Carlo simulations for two-dimensional Ising systems and solving numerically the mean-field equation of motion. We have studied the response behavior for ‘‘positive’’ pulses, where the pulsed field is in the direction of the spontaneous magnetization in the ordered phase. In the disordered phase, of course, this notion is immaterial. One can also study the effect of ‘‘negative’’ pulses on the spontaneous order, where the field direction is opposite the spontaneous magnetization. Although many intriguing features of the domain growth, etc., are expected for such negative pulse problem, we restrict the study here to positive pulses only. We have measured the ratio R_p of the response magnetization (pulse) half width (Δt) to that (δt) of the pulsed external field and the ratio χ_p of the response magnetization peak height (m_p) to the field pulse height (h_p) giving the pulse susceptibility. The temperature variation of these two quantities for various pulse width durations and heights of the external field have been investigated.

We find that for weak pulses, while the width ratio R_p diverges at the order-disorder transition point T_c of the unperturbed Ising system, the pulse susceptibility χ_p does not diverge and for low dimensions, e.g., in the two dimensions studied here, a smeared peak in χ_p occurs at an effective T_c^e, which approaches T_c as the field pulse width δt increases (χ_p diverges at T_c as $\delta t \to \infty$). A mean-field analysis for the absence of the divergence of χ_p for finite δt has been developed. Also, a finite-time scaling analysis, similar to Fisher's finite-size (length) scaling [5], has been developed and compared with the observation of the effective transition temperature T_c^e with the external field pulse width δt.

We have organized this paper as follows. In Sec. II, the model and the simulation techniques have been described. In Sec. III, the results are given. The paper ends with concluding remarks in Sec. IV.

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II. MODEL AND SIMULATION

A ferromagnetically interacting (nearest-neighbor) Ising system in the presence of a time-dependent magnetic field can be described by the Hamiltonian

$$H=-\sum_{i,j} s_i^z s_j^z - h(t)\sum_i s_i^z, \tag{2.1}$$

where the s_i^z's are spin variable having their value ± 1 and $h(t)$ is the time-varying longitudinal magnetic field. Here we have considered the time variation of $h(t)$ as

$$h(t)=\begin{cases} h_p & \text{for } t_0<t<t_0+\delta t \\ 0 & \text{otherwise,}\end{cases} \tag{2.2}$$

where h_p is the amplitude of the field and δt is the duration or the active period of the external field.

In our simulation, we have considered a 500×500 square lattice in two dimensions. At each site of the lattice there is a spin variable $s_i^z=\pm 1$. We update the lattice by stepping sequentially over it, following the Glauber single-spin-flip dynamics. One such full scan over the lattice is a unit time step [Monte Carlo step (MCS)] here. First, we allowed the system to reach equilibrium (at any temperature T) and only after that was the magnetic field $h(t)$ switched on (t_0 is thus much larger than the relaxation time of the system). One can also use random updating sequences. However, it takes more time (MCS) to stabilize the system. We expect all the features of the response studied here to remain qualitatively unchanged for random updating sequences, and we give the results here for sequential updating only. We have measured the maximum height (above the equilibrium value) m_p and half-width Δt of the response magnetization. Here, as mentioned before, the following two quantities have been defined to characterize the response of the system: the pulse width ratio $R_p=\Delta t/\delta t$ and the pulse susceptibility $\chi_p=m_p/h_p$. It may be noted that χ_p reduces to normal static susceptibility as $h_p\to 0$ and $\delta t\to\infty$. At each temperature for fixed h_p and δt, the numerical values of R_p and χ_p are obtained by averaging over 20 random Monte Carlo realizations (initial seed). We have studied the temperature variation of these two quantities. The results are given and discussed in Sec. III. The observation of a finite peak in χ_p at an effective transition temperature T_c^e (which converges to T_c as $\delta t\to\infty$) is analyzed in view of the finite-time scaling behavior, as mentioned before.

To study the similar response in the case where the fluctuations are absent, we have considered the following mean-field dynamical equation of motion for the kinetic Ising system:

$$\tau_0\frac{dm}{dt}=-m+\tanh\left(\frac{m+h(t)}{T}\right). \tag{2.3}$$

Here τ_0 is the microscopic relaxation time, m is the average magnetization (in the mean-field approximation), $h(t)$ is the time-varying pulse field having the same time variation described in Eq. (2.1), and T denotes the temperature. We have solved numerically the above equations using the fourth-order Runge-Kutta method. We have evaluated the pulse width ratio R_p and pulse susceptibility χ_p at various temperatures for fixed pulse width δt and height h_p. The numerical results are given in the next section, where we have compared the results for finite peak height in χ_p (at $T_c=1$) with an approximate analytic estimate of χ_p in such cases.

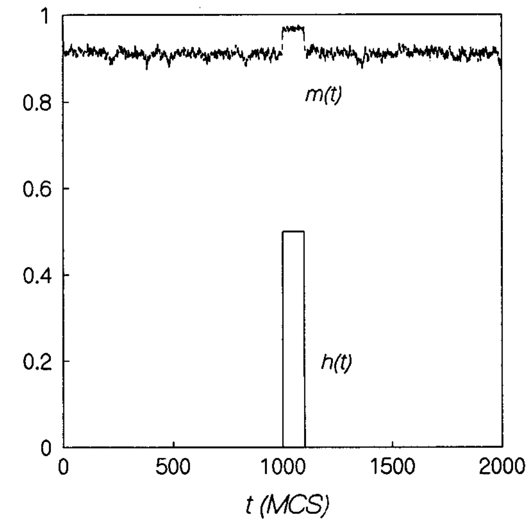

FIG. 1. Time variation of magnetic field $h(t)$ and the response magnetization $m(t)$ in the Monte Carlo case, for $h_p=0.5$ and $\delta t=50$.

III. RESULTS

A. Monte Carlo studies

As mentioned before, our results here are for a two-dimensional Ising system on a square lattice of size 500×500. We applied an external field of amplitude h_p for a duration δt after bringing the system into a steady state. For this, the typical number of Monte Carlo steps required for this size of the lattice chosen here is observed to be of the order of 10^6. The response magnetization has amplitude m_p (measured from the equilibrium value) and a half-width Δt. Figure 1 shows a typical time variation of magnetic field $h(t)$ and the corresponding response magnetization $m(t)$. The dynamical response is characterized by two quantities: the width ratio R_p and the pulse susceptibility χ_p. The temperature variation of these two quantities has been studied. Figure 2 shows the temperature variations of R_p and χ_p for fixed values of h_p ($=0.5$) and for three values of δt ($=5,10,25$ MCS).

Since m_p is bounded from above, a large h_p would saturate m_p and hence χ_p becomes small (due to the saturation). Also, for extremely small h_p, it becomes difficult to identify m_p from the noise and hence the estimate of χ_p becomes erroneous. We found $h_p\cong 0.5$ to be well within the above optimal range. From the figure it is clear that R_p has a sharp divergence at T_R ($\cong 2.30$, somewhat larger than T_c, the Onsager transition temperature, due to the small size of the system) almost irrespective of the values of the pulse width δt. But χ_p shows a peak at different points (significantly above $T_c\sim 2.27$) depending upon the of the field pulse width δt. As the value of δt increases, it is observed that the peak shifts towards T_c from above (and also the peak height grows).

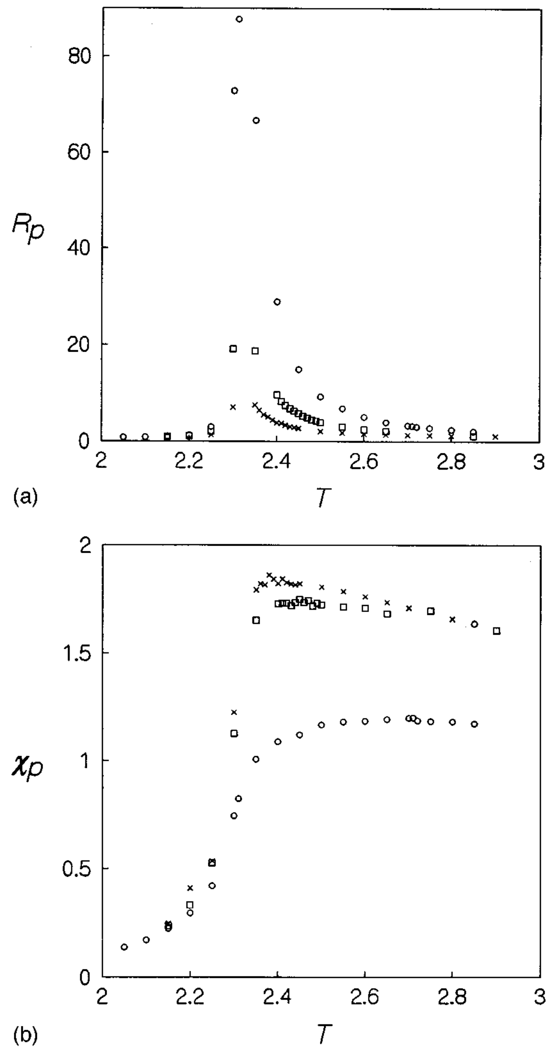

FIG. 2. Temperature variations of (a) R_p and (b) χ_p for different values of δt in the Monte Carlo case. The symbol circle is for $\delta t = 5$, the square is for $\delta t = 10$, and the cross is for $\delta t = 25$.

Let us try to understand why the width ratio R_p diverges at T_c, while the height ratio or pulse susceptibility χ_p shows a peak at some higher value $T_c^e(\delta t)$ depending upon the value of δt: $T_c^e(\delta t) \to T_c$ as $\delta t \to \infty$. The pulselike perturbation probes the response of the system at finite frequencies. Consequently, the χ_p $(=m_p/h_p)$ cannot diverge as the response magnetization height m_p is not an equilibrium value corresponding to the pulse height h_p; rather, the m_p results are bounded by the time window of width δt. On the other hand, the response will take its own relaxation time to come to its equilibrium value (irrespective of the value of δt), when the field is switched off (at $t_0 + \delta t$). This leads to the divergence of Δt, due to a critical slowing down, as T approaches T_c.

The sharp divergence of the width ratio R_p is identified as the consequence of a critical slowing down and the point of divergence is the critical temperature T_c for the ferro-para transition. In fact, since the relaxation after the withdrawal of the pulse is unrestricted by the pulse width, we can assume that $\Delta t \sim \tau \sim |T - T_c|^{-\nu z}$. We therefore plot in Fig. 3 $R_p^{-1/\nu z}$ versus T and find a straight-line plot with $\nu z \cong 2.0$ in the two-dimensional case. This compares well with the previous estimates of the value of νz [6].

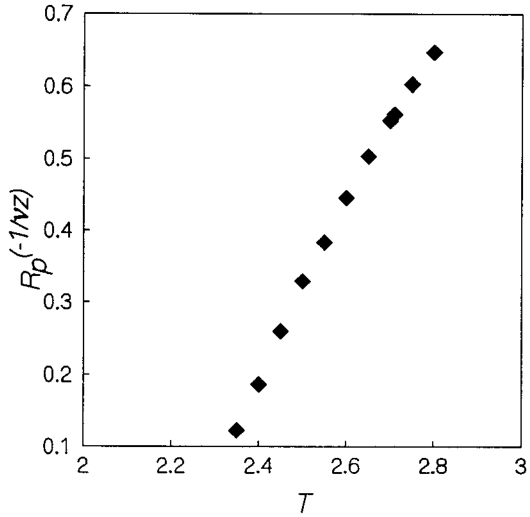

FIG. 3. Variation of $R_p^{-1/\nu z}$ against T, with $\nu z = 2$.

As the growth of the height of the magnetization response (and its maximum value m_p) is very much bounded by the time window δt of the applied field pulse, the anomalous behavior of χ_p [having a finite peak at a shifted temperature $T_c^e(\delta t)$] may be considered to be due to the finite-size (in time) effect. Similar to the finite-size (in length) scaling theory of Fisher [5], where a finite-size system shows an effective (nonsingular or nondivergent) pseudocritical behavior at $T_c^e(L)$ when the correlation length ξ becomes of the order of the system size L, we suggest a finite-time (δt) scaling behavior here for χ_p. If the relaxation time $\tau \sim \xi^z \sim |T - T_c|^{-\nu z}$, where ν is the correlation length exponent and z is the dynamic exponent, then χ_p would show a peak at the temperature T_c^e, here when $\tau(T_c^e) \sim \delta t$, $|T_c^e(\delta t) - T_c|^{-\nu z} \sim \delta t$, or

$$T_c^e(\delta t) \sim T_c + C(\delta t)^{-1/\nu z}, \tag{3.1}$$

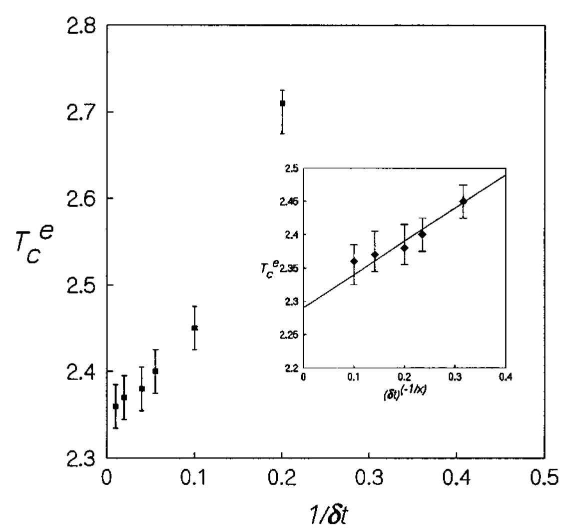

FIG. 4. Variation of T_c^e with respect to $1/\delta t$ in the Monte Carlo case. The inset shows the variation of T_c^e with respect to $(\delta t)^{-1/x}$, $x = 2$.

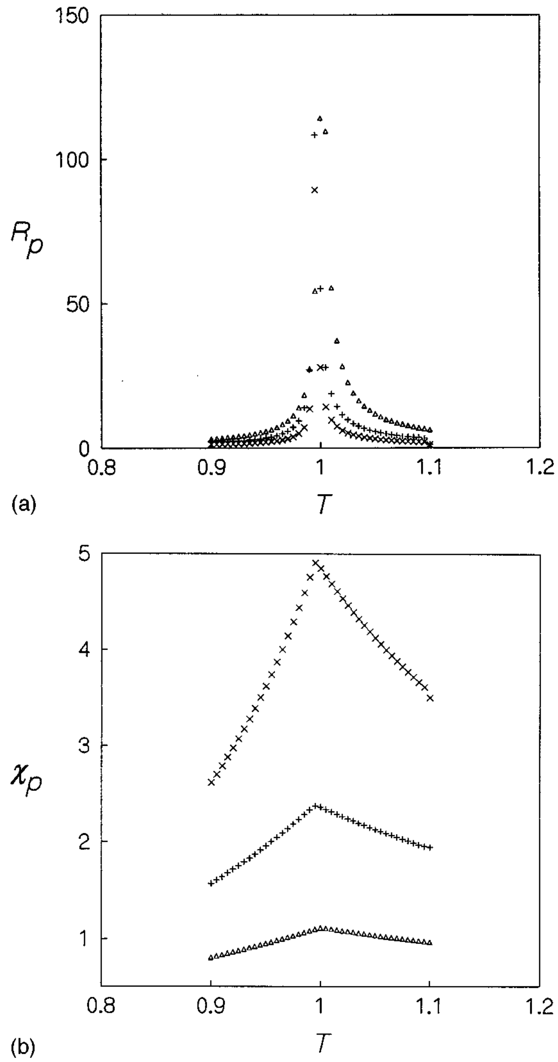

FIG. 5. Temperature variation of (a) R_p and (b) χ_p for different values of δt in the mean-field case: the triangle is for $\delta t\ =\ 8$, the plus is for $\delta t\ =\ 16$, and the cross is for $\delta t\ =\ 32$.

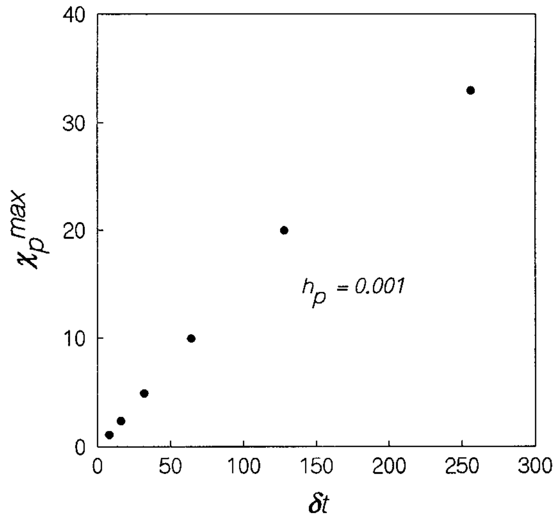

FIG. 6. Variation of $\chi_p^{\max}$ with respect to δt in the mean-field case.

where C is some constant. In fact, Fig. 4 shows that the effective peak position T_c^e indeed approaches T_c as $\delta t \rightarrow \infty$. The inset shows the plot of T_c^e with $(\delta t)^{-1/x}$, which gives a straight line for $x = \nu z \cong 2.0$. This again suggests $\nu z \cong 2.0$ and also the extrapolated value of T_c becomes about 2.29, which compares well with the Onsager value, comparable to the previous estimate.

B. Mean-field results

We have solved the mean-field equation for the response magnetization (2.3) using the fourth-order Runge-Kutta method. Figure 1 shows the typical variation of the response magnetization and field. Here also we have measured the width ratio and the pulse susceptibility and studied the temperature variation of these two quantities.

Figure 5 shows the temperature variation of R_p and χ_p for different values of δt. R_p diverges and χ_p peaks at the same order-disorder transition point ($T_c = 1$ here). We have also studied the variation of the maximum value of χ_p ($\chi_p^{\max}$ at $T = T_c$) with respect to the duration of the pulsed field. It may also be mentioned that the peak height was found to increase with increasing pulse width δt, and χ_p diverges as $\delta t \rightarrow \infty$: $\chi_p^{\max} \sim (\delta t)^y$, $y \cong 1.0$. Figure 6 shows the variation of $\chi_p^{\max}$ with δt.

In order to comprehend these observations, we solve Eq. (2.3) in a linearized limit (large T and small h_p; specifically $T > 1$, $h_p \rightarrow 0$). In such a limit, the equation of motion becomes

$$\tau_0 \frac{dm}{dt} = -\epsilon m + h(t)/T, \quad \epsilon = (T-1)/T. \tag{3.2}$$

One can solve Eq. (3.2) using the form $m(t) = m_0 e^{-t/\tau}$, which gives

$$\tau_0 \frac{dm_0}{dt} e^{-t/\tau} - \frac{\tau_0}{\tau} m_0 e^{-t/\tau} = -\epsilon m_0 e^{-t/\tau} + \frac{h(t)}{T}, \tag{3.3}$$

giving $\tau/\tau_0 = \epsilon^{-1}$ and

$$\tau_0 \frac{dm_0}{dt} e^{-t/\tau} = h(t)/T. \tag{3.4}$$

Integrating Eq. (3.4) for $h(t) = h_p$ for a finite time width δt and $h(t) = 0$ otherwise, one gets $m_p \sim h_p \delta t / T_c \tau_0$ at $T = T_c = 1$(when $\tau \rightarrow \infty$). This gives

$$\chi_p^{\max} = \chi_p(T_c) = m_p(T_c)/h_p \sim \delta t / T_c \tau_0 \sim \delta t / \tau_0 . \tag{3.5}$$

We have checked the above linear relationship of $\chi_p^{\max}$ with δt for extremely small values of the pulse field amplitude h_p (see Fig. 6).

IV. SUMMARY

We have studied the Glauber (order-parameter-nonconserving) dynamics of an Ising system under a time-varying external magnetic field when the field is applied as a pulse of finite time width, after the system reaches equilibrium. The time variation of the response magnetization is studied as a function of pulse width δt, height h_p, and temperature T of the system. We have measured specifically $R_p = \Delta t / \delta t$ and $\chi_p = m_p / h_p$, where Δt is the time width of the response magnetization and m_p is the maximum height of the response magnetization above its equilibrium value.

Our computer simulation results for the square lattice show $R_p \sim |T_c - T|^{-\nu z}$, with $T_c \cong 2.30$, the Onsagar value, and χ_p has a peak $\chi_p^{\max}$ at $T_c^e > T_c$, such that a finite-size (in time) scaling behavior is observed: $T_c^e = T_c + C(\delta t)^{-1/\nu z}$, with $\nu z \cong 2.0$. The numerical solutions of the mean-field Eq. (2.3) showed $R_p \sim 1/(T-1)$ and the pulse susceptibility peak value $\chi_p^{\max} \sim \delta t$, occurring at $T = T_c = 1$. Theoretical analyses of the finite-size (in time) scaling behavior (of T_c^e in the Monte Carlo case) and of the peak $\chi_p^{\max}(\delta t)$ (in the mean-field case) are also made.

ACKNOWLEDGMENTS

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[1] See, e.g., *Application of the Monte Carlo Method in Statistical Physics*, edited by K. Binder (Springer, Heidelberg, 1984); *Annual Reviews of Computational Physics*, edited by D. Stauffer (World-Scientific, Singapore, 1994), Vol. 1.

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